Superfluidity in One Dimension as a Dynamical Phenomenon

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Thank you to Junko Taniguchi & Masaru Suzuki
(U. Electro-Communications, Tokyo)
Criterion for Superfluidity

Landau’s criterion
\[
\min \left\{ \frac{\epsilon(p)}{p} \right\} = \nu_{\text{Landau}} > 0
\]

⇒ how to understand finite \( T \) ? etc.

Helicity modulus [ME Fisher et al, 1973]

\[
\hat{\Psi}(x + L, y, z) = e^{i\varphi} \hat{\Psi}(x, y, z)
\]

\[
\Upsilon(T) = \lim_{L \to +\infty} \frac{L}{S} \left( \frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right)_{\varphi=0}
\]

\[
\Upsilon(T) = \frac{\hbar^2}{m} \rho_s(T) \quad \text{Superfluid density}
\]
Superfluidity in 2D

No off-diagonal LRO at $T>0$ (Mermin-Wagner theorem)

But helicity modulus is finite for $T<T_{\text{BKT}}$

“universal jump” at $T<T_{\text{BKT}}$ [Nelson-Kosterlitz 1977]

Superfluidity is indeed observed in torsional oscillator measurements of 2D $^4$He film [Bishop-Reppy 1978]

Dynamical effects are also important [Ambegaokar-Halperin-Nelson-Siggia 1978]
Superfluidity in 1D?

Helicity modulus vanishes in 1D (in thermodynamic limit)

\[ \gamma_{1D}(T) = \lim_{L \to +\infty} L \left( \frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \bigg|_{\varphi=0} = 0 \]

Hence, no superfluidity in 1D?
Liquid $^4$He in 1D nanopore

"FSM-16"

channel length: 0.2～0.5 μm

length/diameter ~ 100

[Taniguchi-Aoki-Suzuki 2010]
Results (2.8nm diameter)

Superfluid(-like) response!

superfluidity suppressed at higher pressures

Dissipation peak at “superfluid transition temperature”
Phase Diagram

[Taniguchi-Aoki-Suzuki 2010]
cf.) $^4$He in 3D porous media

Gelsil (pore $\Phi \sim 25\,\text{Å}$)

Shirahama et al. 2004

Similar-looking phenomena but different physics (3D LRO)

Eggel, M.O. - Shirahama 2011
Quantum Monte Carlo simulation (Worm Algorithm-Path Integral) of microscopic Hamiltonian for 4He in 1D nanopore

Quantitative agreement with TLL on static quantities (Del Maestro-Affleck 2010, Del Maestro-Boninsegni-Affleck 2011)

But not (yet) for the diameter 2.8nm of Taniguchi et al. expt.
Finite-size effect?

Helicity modulus $\Upsilon(T)$ of a 1D system vanishes, but only in the thermodynamic limit

maximum onset temperature of helicity modulus
$$\frac{\epsilon_0}{k_B} = \frac{\hbar v K}{L} < \frac{\hbar^2 \pi \rho_0}{mL} \approx 0.2K$$

Too low to account the experimental results
(onset temperature can be $\sim 1K$ or higher)

Tomonaga-Luttinger Liquid

$$\Upsilon(T, L) = \Upsilon_0 \left(1 + \frac{\epsilon_0 \vartheta_3'(0, e^{-2\epsilon_0/T})}{T \vartheta_3(0, e^{-2\beta\epsilon_0})}\right)$$

Yamashita-Hirashima 2009
Why superfluidity in 1D?

finite-size effect

\[
\gamma_{1D}(T) = \lim_{L \to +\infty} L \left( \frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \bigg|_{\varphi=0} = 0
\]

\[
\gamma(T) \overset{?}{=} \frac{\hbar^2}{m} \rho_s(T)
\]

Static property in equilibrium

Dynamics
What is Superfluidity?

How will the fluid behave?
- eventually come to rest (normal fluid)
- move perpetually at velocity $v$ (superfluid)

container wall
in equilibrium at velocity $v$
What is superfluidity?

Initial condition of the fluid: Galilean boost of the equilibrium fluid at rest, with velocity $v$

$$\rho_{\text{ini}} = e^{-i\hbar mvx} \rho_{\text{eq}} e^{i\hbar mvx}$$

$$\langle \mathcal{O}(t) \rangle = \text{Tr} \left( e^{i\mathcal{H}t/\hbar} \mathcal{O} e^{-i\mathcal{H}t/\hbar} \rho_{\text{ini}} \right) = \text{Tr} \left( e^{i\tilde{\mathcal{H}}t/\hbar} \tilde{\mathcal{O}} e^{-i\tilde{\mathcal{H}}t/\hbar} \rho_{\text{eq}} \right)$$

$$\tilde{\mathcal{O}} \equiv e^{i\hbar mvx} \mathcal{O} e^{-i\hbar mvx}$$

$$\tilde{\mathcal{H}} = \sum_i \left( \frac{\hbar^2}{2m} (\vec{p}_i + m\vec{v})^2 + U(\vec{r}_i) \right) + \sum_{i>j} V(\vec{r}_i - \vec{r}_j)$$

“effective Hamiltonian” equivalent to phase twist
What happens at $t \to \infty$?

Fluid reaches equilibrium with respect to effective Hamiltonian (in the presence of static wall potential)

In a normal liquid, the resulting state should be equivalent to $\rho_{eq}$. But in a superfluid, a fraction of fluid is still moving at velocity $v$

Free energy density

$$f(\vec{v}) \sim f(\vec{0}) + \frac{\rho_s}{2} \vec{v}^2$$

$$\gamma(T) = \lim_{L \to +\infty} \frac{L}{S} \left( \frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \bigg|_{\varphi=0} = \frac{\hbar^2}{m} \rho_s(T)$$

Helicity modulus = Superfluid density?
What is assumed?

Fluid reaches equilibrium with respect to effective Hamiltonian (in the presence of static wall potential) i.e. we need (hidden) assumption of thermalization of in order to derive $\Upsilon = \rho_s$

Integrable systems in 1D: thermalization is absent due to infinite # of conserved quantities, so the equivalence between $\Upsilon$ and $\rho_s$ would break down
Generic Systems in 1D?

“Non-integrable models thermalize”
- common belief

This may not be always the case, but we would assume that realistic, generic non-integrable systems eventually thermalize

\[ \Rightarrow \text{resurrection of } \rho_s(T) = \frac{\hbar^2}{m} \rho_s(T) \]

Then the superfluidity is absent in 1D in the strict sense. However, due to the anomalous dynamics in 1D, the approach to equilibrium could be very slow.

Superfluidity might be observed at experimentally relevant timescale
Phase Slips

Decay of “superflow” and thermalization caused by phase slips

**Thermal Phase Slips**

\[ \Gamma_{TPS} \sim \exp \left( -\frac{\Delta F}{k_B T} \right) \]

**“Quantum” Phase Slips**
[Khlebnikov 2005]

\[ \Gamma_{QPS} \sim \exp \left( -\frac{\hbar \pi \nu \rho_0}{k_B T} \right) \]

Exponentially suppressed PS rate at low T: manifestation of constrained dynamics in 1D but cannot account the experimental results on 1D $^4$He
Required Formulation

- Include quantum&thermal fluctuations beyond the leading exponential
- Include explicitly the potential due to the container wall
  (in $D \geq 2$ the wall effect can be replaced by a \textit{boundary condition}, but \textbf{NOT in 1D})
- Include the interaction among particles ($^4$He atoms)
- Take the conserved (or nearly conserved) quantities into account properly
- Consider finite-frequency response

\textbf{Memory-matrix formulation based on TL Liquid theory}

\textit{cf.}) conductivity [Rosch-Andrei 2000]
What to calculate?

(Total) Momentum Response Function

\[
\chi(t) = -\frac{i}{\hbar} \theta(t) \langle [\Pi(t), \Pi(0)] \rangle
\]

\[
\Pi = \sum_j p_j
\]

measures the response of the system to the perturbation in the effective Hamiltonian

\[
\tilde{H} = \sum_i \left( \frac{\hbar^2}{2m} (\vec{p}_i + m\vec{v})^2 + U(\vec{r}_i) \right) + \sum_{i>j} V(\vec{r}_i - \vec{r}_j)
\]

normal fluid density \[
\rho_n = -\frac{1}{m} \lim_{\omega \to 0} \chi(\omega)
\]
Tomonaga-Luttinger Liquid

\[ \mathcal{H}_* = \frac{\hbar v}{2\pi} \int dx \left[ \frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \theta)^2 \right] \]

Low-energy fixed point with \( \infty \) number of conserved qtns

\[ J = \frac{mvK}{\pi} \int dx \partial_x \theta(x, t) \quad \text{particle mass current} \]

\[ P = \frac{\hbar}{\pi} \int dx \partial_x \phi \partial_x \theta \quad \text{energy current} \]

Due to the curvature of the dispersion, total momentum is

\[ \Pi = J + \frac{vKm}{\hbar \pi \rho_0} P \]
Wall Potential

We assume periodic potential due to the wall (reasonable for FEM-16 expt)

\[ H_{PS} = \sum_{n>0,m} \frac{\hbar v g_{nm}}{\pi a_0^2} \int dx \cos (2n\phi(x) + \Delta k_{nm}x). \]

“irrelevant” in the RG sense, but is important since it causes phase slips

\( J \) and \( P \) (and thus \( \Pi \)) are exactly conserved in pure TLL (= fixed point Hamiltonian \( H^* \)), but not conserved in the presence of \( H_{PS} \)

Nevertheless, the decay is slow due to constrained dynamics in 1D -- how to describe?
Memory Matrix Formalism

\[ \chi(\omega) = \text{Tr} \left\{ V [\omega \hat{1} + i \hat{M}(\omega)]^{-1} i \hat{M}(\omega) \hat{\chi}(\omega) \right\} \]

\[ \hat{\chi} \sim \text{diag}\{\chi_{JJ}, \chi_{PP}\} \]

\[ \hat{M} \quad : 2x2 \text{ matrix describing the decay rates of two currents} \]

Perturbative evaluation in \( H_{PS} \)

Results

\[ \omega = \omega_0 = 2 \, \text{kHz} \]

K = 4.2  \quad K = 9.2

Expt. [Taniguchi et al. 2010]
Double onset

Large incommensurability: $\Delta k_{10} = 0.5a_0^{-1}$

$K=6.2$

$\Delta k_{10} = 0.001a_0^{-1}$
Frequency Dependence

\[ \omega = 10^{-2} \omega_0 \]

\[ \omega = 10^2 \omega_0 \]

\[ T_p \sim \omega \frac{1}{2K - 3} \]

cf.) Zaikin et al. (1997)
Lobos-Giamarchi (2005)
Danshita-Polkovnikov (2011)
Frequency dependence (expt.)

J. Taniguchi et al. (private communications)

500Hz vs. 2000Hz

Pressure effect
Frequency dependence (expt.)

J. Taniguchi et al. (private communications)

\[
\frac{T_{ph}}{T_{pl}} \propto \left( \frac{2000 \text{ Hz}}{500 \text{ Hz}} \right)^{\frac{1}{2K-3}}
\]

\[
T_p \sim \omega^{\frac{1}{2K-3}}
\]

may be explained by the pressure dependence of the Luttinger parameter?
Strongly Inhibited Transport of a Degenerate 1D Bose Gas in a Lattice

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FIG. 1. Damped oscillations of a 1D Bose gas in an optical lattice. Shown are plots of velocity versus wait time $t_w$ from $t_w = 0$ to 110 ms, and for axial lattice depths of (a) $0E_R$, (b) $0.2E_R$, (c) $0.6E_R$, and (d) $1.0E_R$. The oscillations are qualitatively similar to those of a harmonic oscillator, but with amplitudes that decay with $t_w$.
Relevance to cold atoms

[Tokuno-Giamarchi 2011]

Frequency dependence may be probed over a wider range, than in torsional oscillator measurements of $^4$He
Relevance to “supersolid”?

Skew dislocations in solid 4He behaves as TLL

[Boninsegni et al. 2007]

Dislocation network (“Shevchenko state”)

[Balibar 2010]
Conclusions

- Helicity modulus in 1D vanishes (in thermodynamic limit)
- Superfluidity in 1D is essentially dynamical phenomenon, related to absence of (or anomalously slow) thermalization
- “Superfluid density” dependence on probe frequency is predicted
- Momentum response couples to 2 conserved currents in TLL / conservation broken by wall potential
- Qualitative agreement with 4He in 1D nanopore
- Possible relevance to dislocations in solid 4He, and to cold atoms